

Engineering Statistics

IES 302

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8 Discrete Random Variable

Flip an unfair coin 10 times. (The probability of getting head for each time is 0.3.)
 Count the number of heads.

Random variable $X \sim \text{binomial}(10, 0.3)$

```
>> X = binornd(10,0.3)
```

X =

3

Again, flip an unfair coin 10 times.

```
>> X = binornd(10,0.3)
```

X =

2

Again, flip an unfair coin 10 times.

```
>> X = binornd(10,0.3)
```

X =

2

Again, flip an unfair coin 10 times.

```
>> X = binornd(10,0.3)
```

X =

5

Again, flip an unfair coin 10 times.

```
>> X = binornd(10,0.3)
```

X =

1

Again, flip an unfair coin 10 times.

```
>> X = binornd(10,0.3)
```

X =

4

```
>> X = binornd(10,0.3,20,10)
```

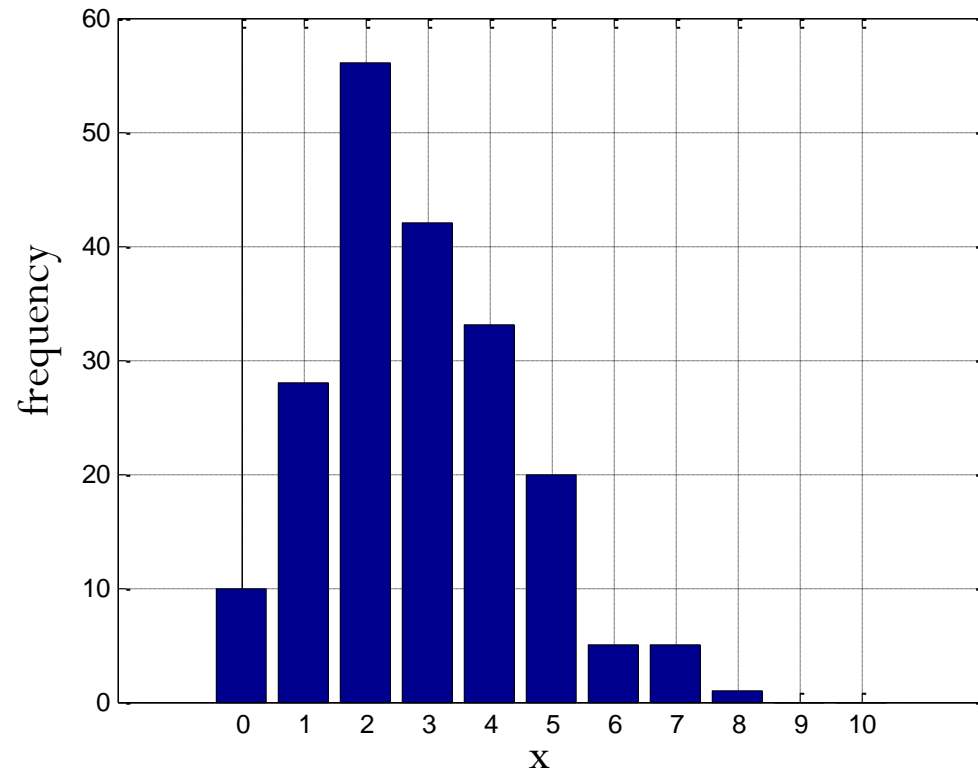
X =

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 4 | 4 | 5 | 7 | 2 | 2 | 2 | 2 | 1 |
| 3 | 5 | 3 | 1 | 0 | 4 | 2 | 1 | 2 | 3 |
| 5 | 2 | 2 | 6 | 4 | 2 | 2 | 4 | 3 | 1 |
| 1 | 2 | 2 | 4 | 2 | 4 | 3 | 3 | 3 | 5 |
| 4 | 1 | 4 | 3 | 3 | 4 | 2 | 2 | 2 | 2 |
| 2 | 1 | 3 | 1 | 5 | 2 | 5 | 2 | 1 | 2 |
| 4 | 0 | 3 | 3 | 2 | 1 | 2 | 1 | 3 | 1 |
| 4 | 4 | 0 | 2 | 3 | 6 | 2 | 3 | 1 | 1 |
| 5 | 0 | 3 | 3 | 7 | 1 | 3 | 1 | 3 | 8 |
| 1 | 2 | 4 | 4 | 1 | 5 | 2 | 4 | 5 | 1 |
| 5 | 2 | 4 | 6 | 3 | 2 | 3 | 3 | 5 | 0 |
| 2 | 4 | 0 | 0 | 2 | 2 | 3 | 2 | 0 | 2 |
| 4 | 3 | 3 | 2 | 2 | 2 | 1 | 2 | 7 | 4 |
| 2 | 4 | 2 | 1 | 3 | 3 | 4 | 3 | 5 | 2 |
| 5 | 3 | 2 | 3 | 4 | 2 | 3 | 3 | 1 | 2 |
| 2 | 6 | 2 | 3 | 4 | 4 | 4 | 5 | 6 | 7 |
| 5 | 1 | 2 | 4 | 3 | 3 | 0 | 5 | 0 | 2 |
| 1 | 4 | 1 | 3 | 1 | 4 | 2 | 4 | 2 | 4 |
| 5 | 2 | 2 | 3 | 3 | 5 | 3 | 5 | 2 | 1 |
| 4 | 2 | 4 | 3 | 2 | 5 | 7 | 2 | 3 | 1 |

Generate X 200 times. Put the results in a table of size 20x10



Histogram

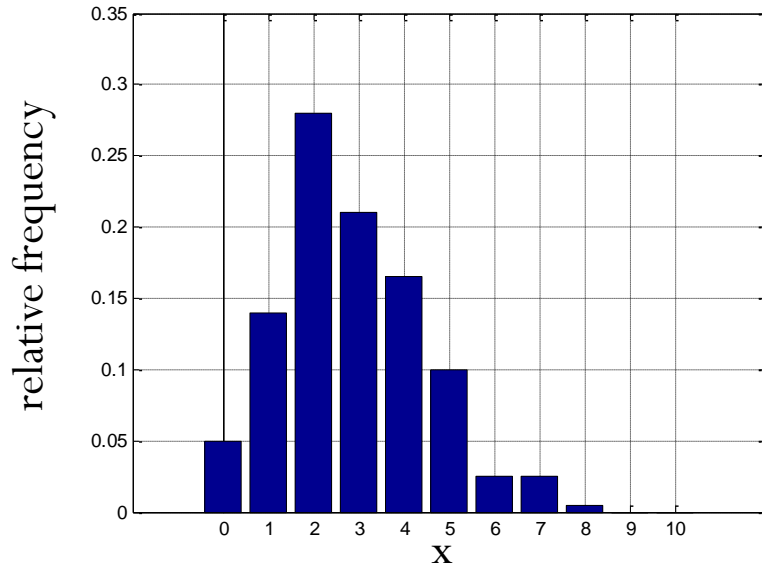


```
[N, x] = hist(reshape(X,1,prod(size(X))),0:10)
```

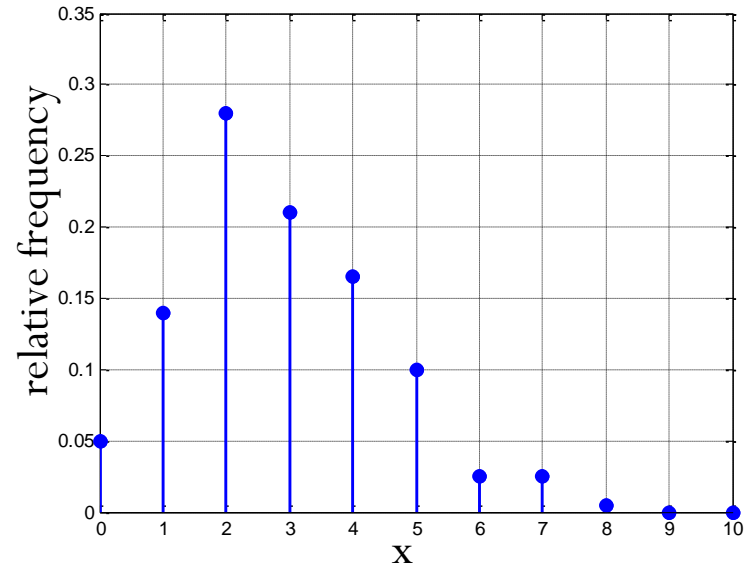
```
bar(x,N)
```

```
Grid on
```

Relative Frequency



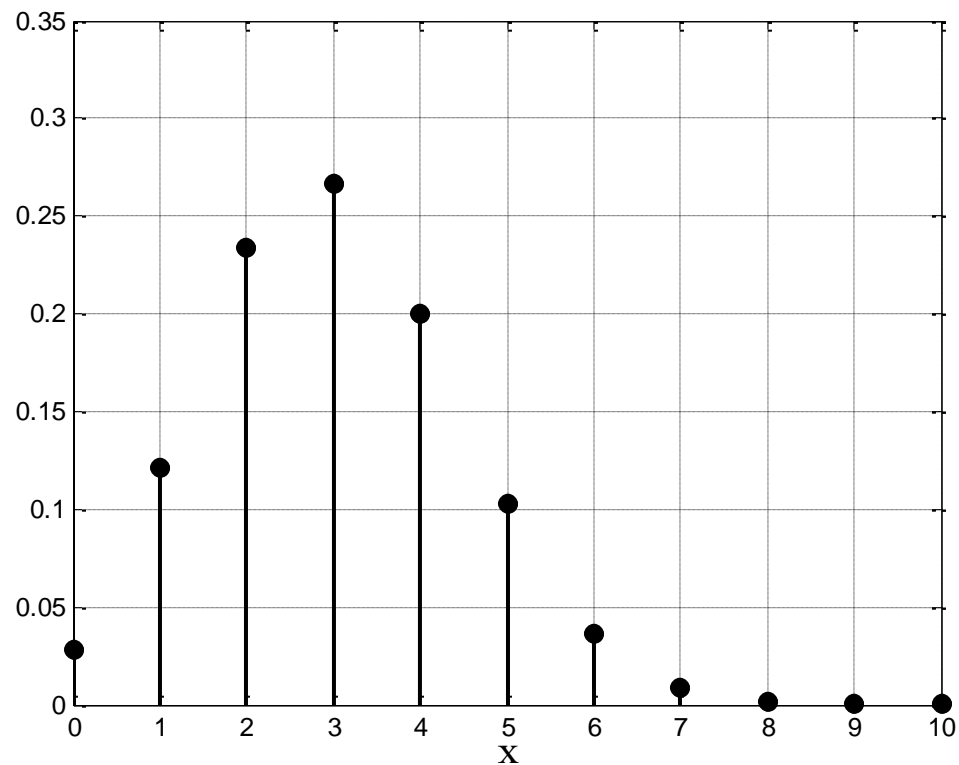
```
rf = N/prod(size(X))  
bar(x,rf)  
grid on
```



```
stem(x,rf,'filled','LineWidth',1.5)  
grid on
```

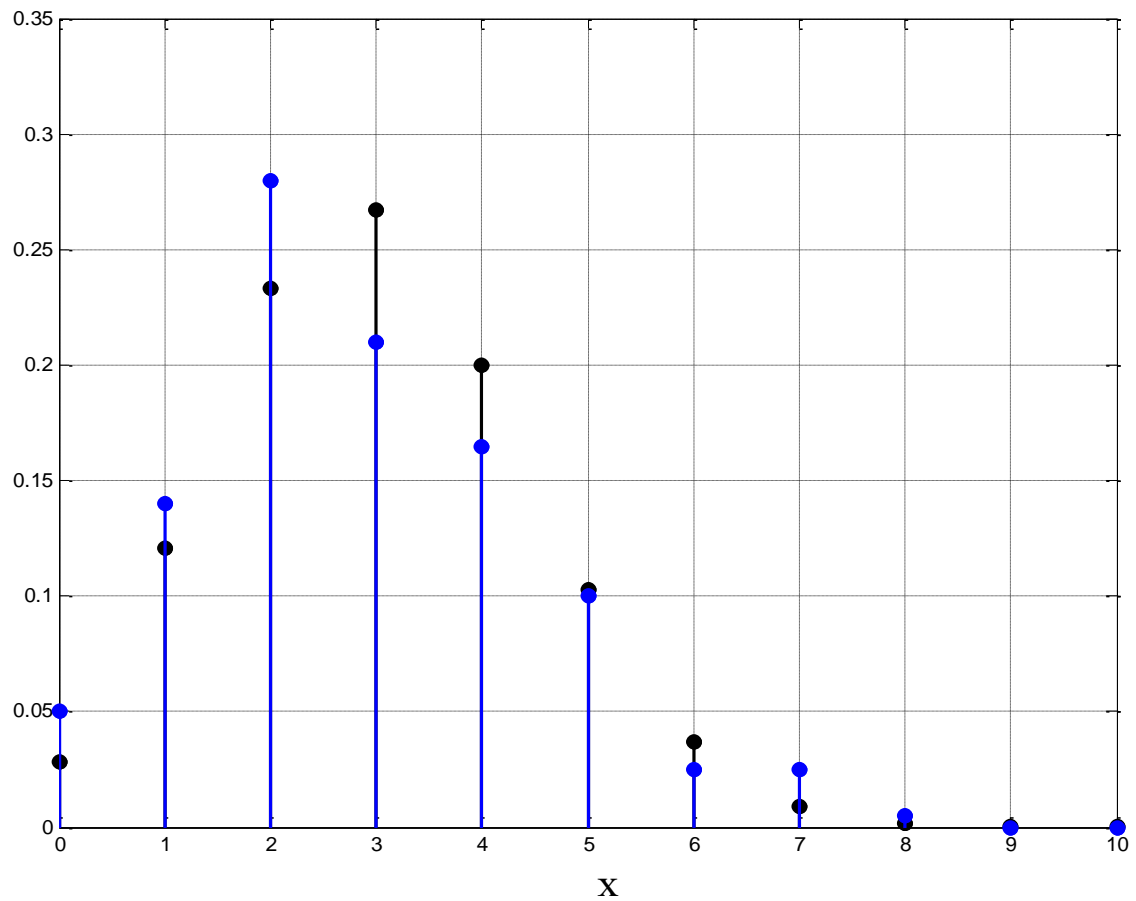
pmf for X

$$p_X(x) = \binom{10}{x} 0.3^x (1 - 0.3)^{10-x}$$



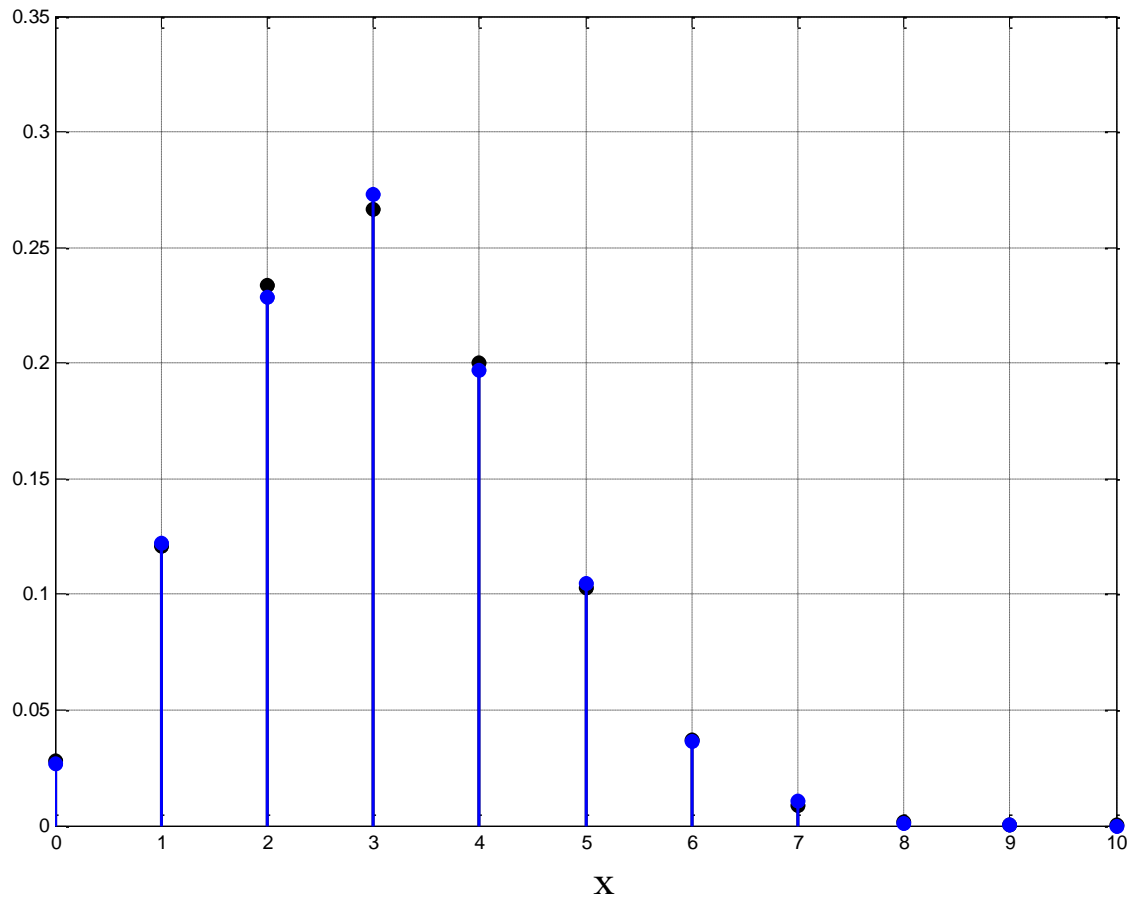
```
p = binopdf(x,10,0.3)  
stem(x,p,'k','filled','LineWidth',1.5); grid on
```

Comparison



With larger number of samples

`X = binornd(10,0.3,100,100);`



Calculations of Expected Values



sum exp(-alpha) k * alpha^k / k! from k =0 to infinity



Examples Random

Infinite sum:

$$\sum_{k=0}^{\infty} \frac{k \exp(-\alpha) \alpha^k}{k!} = \alpha \quad \mathbb{E}X = \alpha$$

n! is the factorial function »

Poisson(α)

sum k * n!/(k!(n-k)!) * p^k * (1-p)^(n-k) from k =0 to n



Examples Random

Sum:

$$\sum_{k=0}^n \frac{k n! p^k (1-p)^{n-k}}{k! (n-k)!} = n p \quad \mathbb{E}X = n p$$

n! is the factorial function »

Binomial(n,p)

Computation of $\mathbb{E}[X^2]$



sum exp(-alpha) k^2 * alpha^k / k! from k =0 to infinity



Examples Random

Infinite sum:

$$\sum_{k=0}^{\infty} \frac{k^2 \exp(-\alpha) \alpha^k}{k!} = \alpha (\alpha + 1)$$

$$\mathbb{E}[X^2] = \alpha + \alpha^2$$

n! is the factorial function >

Poisson(α)

sum k^2 * n!/(k!(n-k)!) * p^k * (1-p)^(n-k) from k =0 to n



Examples Random

Sum:

$$\sum_{k=0}^n \frac{k^2 n! p^k (1-p)^{n-k}}{k! (n-k)!} = p (n^2 p - n p + n)$$

$$\mathbb{E}[X^2] = np(1-p) + (np)^2$$

n! is the factorial function >

Binomial(n,p)

One number to represent a data set

$$\begin{aligned} \text{data} &= [X_1, X_2, X_3, X_4, X_5, X_6, X_7] \\ &= [-10, -9, 1, 2, 3, 4, 5] \end{aligned}$$

Find one number
(a) to represent
this data set.

